Nuclear Reactor Core Reloading Optimization

by Using Nonlinear Optimization and Search Heuristics

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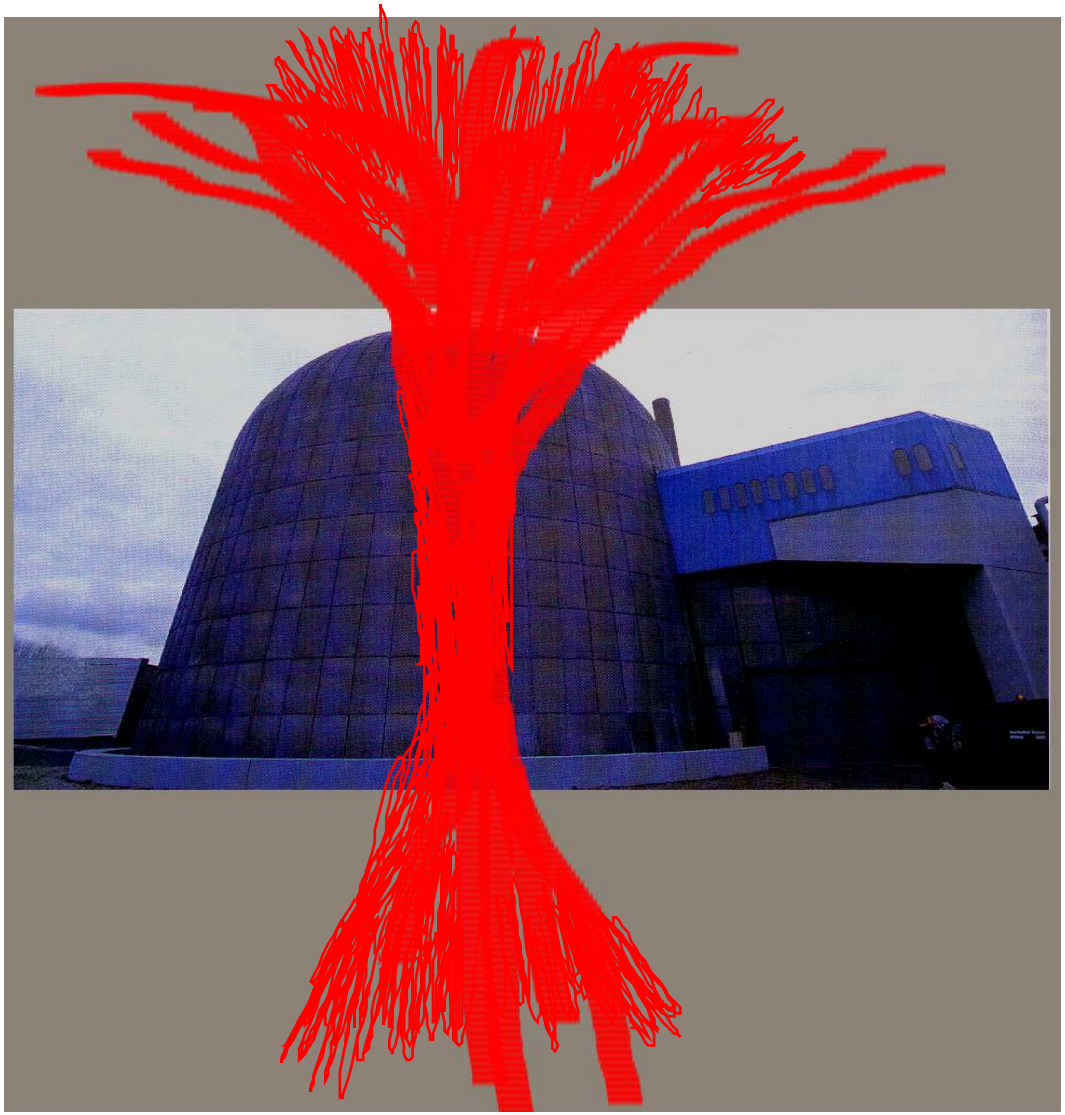
Outline

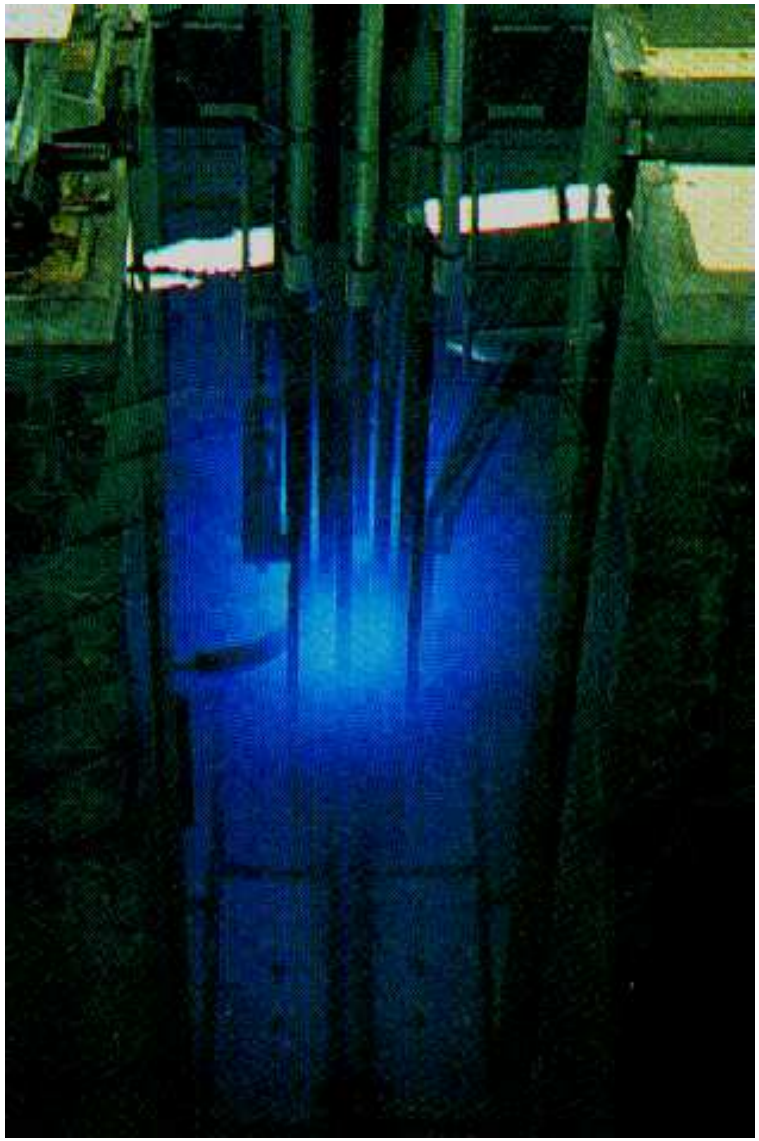
1. Nuclear reactor;
2. Introduction;
3. Physical Background;
   Partial Differential Equation;
4. Mathematical Optimization Problem;
5. Solution Strategies;
6. Some Results,;
7. Extensions: Discretization, Burnable Poisons;
8. More Exact Model to Handle Fractional Solutions;
9. Results Including BP Optimization;
10. Conclusions and Further Steps.

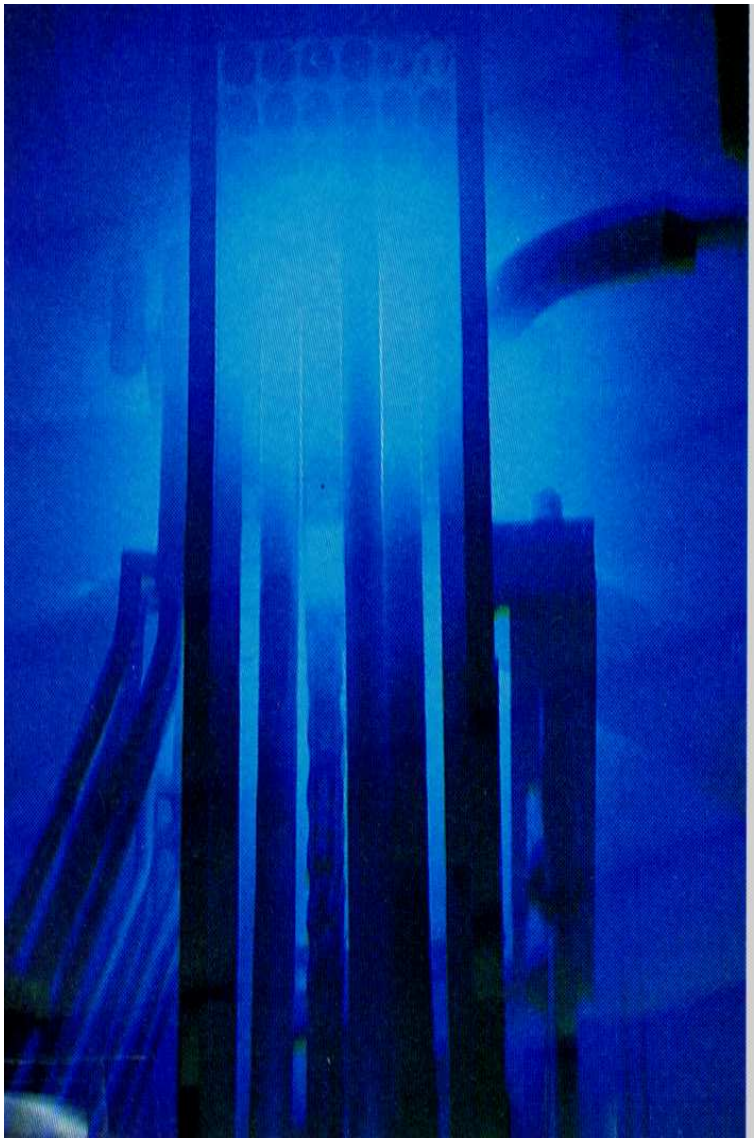
Nuclear Reactor

Bad Fuel Management



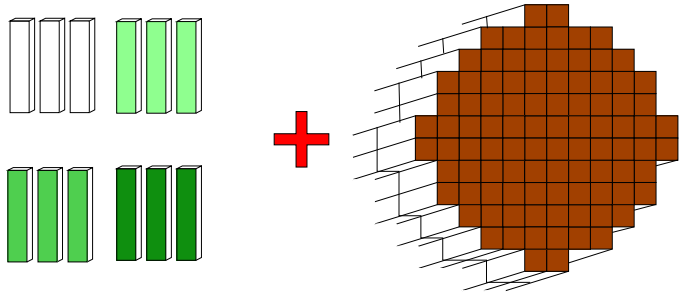
Nuclear Reactor Core – Bundles in the Pool



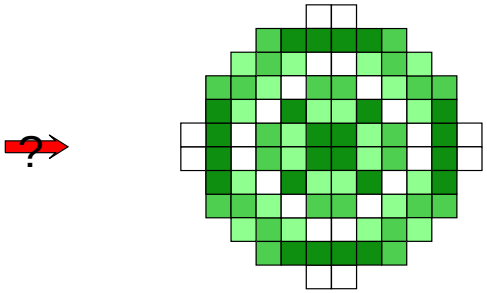


Introduction

Given a reactor core and fuel bundles of different ages ...
... assign bundles to nodes such that



1. the yield is maximized;
2. physical side constraints are satisfied;
3. safety limitations are respected.

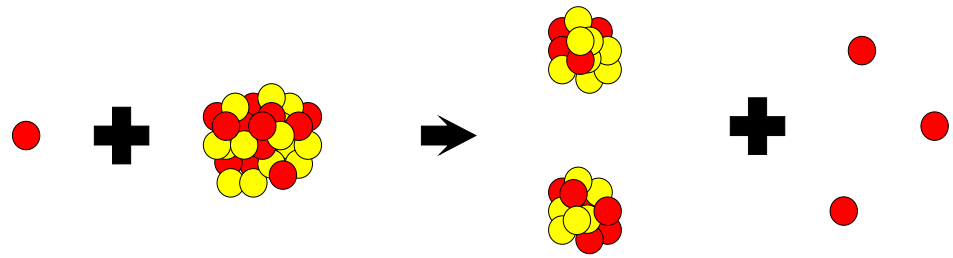


→ **Assignment problem; objective is described by a PDE;**
→ **Nonlinear mixed-integer optimization problem.**

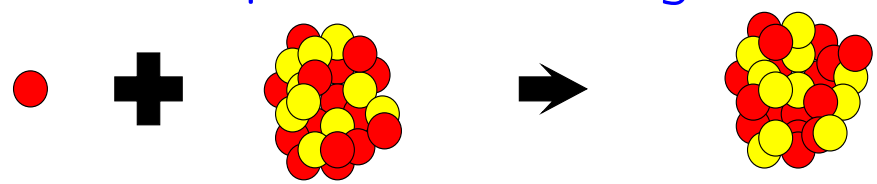
Physical Background

Reactivity in the core is determined by neutron flows.

Neutrons density increases due to fission



and decreases due to absorption and leakage.



The neutron flow is described by the diffusion equation:

$$\frac{\partial \phi_{x,t}}{\partial t} - D\nabla_x^2 \phi_{x,t} + \Omega_x^a \phi_{x,t} = \Omega_{x,t}^f \phi_{x,t}$$

where ϕ is the neutron flux; Ω^a the neutron absorption rate;
D the diffusion coefficient; Ω^f the neutron production rate.
$\Omega^{a/f}$ depend on the fuel composition.

Physical Background (2)

Under some assumptions,
the diffusion equation can be replaced by the system

$$\frac{\partial k^{\infty}_{x,t}}{\partial t} = -\sigma k^{\infty}_{x,t}\phi_{x,t}. \tag{1}$$

$$-D\nabla^2_x\phi_{x,t} + \phi_{x,t} = \frac{1}{k^{eff}(t)}k^{\infty}_{x,t}\phi_{x,t}. \tag{2}$$

Here

- σ is a constant describing the change in fuel composition;
- $k^{\infty} = \Omega^f/\Omega^a$ is the infinite multiplication factor, which describes
$$\frac{\text{local neutron production rate}}{\text{local neutron loss rate}};$$
- k^{eff} behaves as the inverse of an eigenvalue, and can be viewed as
$$\frac{\text{total neutron production rate}}{\text{total neutron loss rate}}.$$

Physical Background (3)

Equation (2) is a Helmholtz equation. In 1D, it is a Sturm-Liouville differential equation, which has nice properties:

- All eigenvalues are real and simple;
- All eigenvectors are real;
- The smallest eigenvalue has nonnegative eigenvector;

Moreover, an adjoint equation exist: given the DE

$$L[\phi(x)] \;=\; \lambda k^{\infty}_{x,t}\phi_{x,t})$$

with boundary condition

$$\phi(x) \;=\; 0$$

there exists a *Greens function* $g(x,s)$ such that

$$\phi(x) \;=\; \int_X g(x,s)\lambda k^{\infty}_{s,t}\phi_{s,t}ds. \tag{3}$$

Fixed Core Model

This allows a coarse discretization grid, with one grid-point per bundle. Then the Greens function equation (3) becomes a matrix eigenvalue equation:

$$k_t^{eff}\phi_t \;=\; G(k_t^{\infty}\cdot\phi_t) \quad \forall t, \tag{4}$$

where $k_t^{\infty} \;=\; (k_{1,t}^{\infty},...,k_{N,t}^{\infty})$
$\phi_t \;=\; (\phi_{1,t},...,\phi_{N,t})$.

Using the normalization

$$k_t^{\infty T}\cdot\phi_t \;=\; 1 \qquad \forall t \tag{5}$$

and a discretization of the burnup (1)

$$k_{t+1}^{\infty} - k_t^{\infty} \;=\; -\,\sigma\Delta t\, k_t^{\infty}\cdot\phi_t \quad \forall t, \tag{6}$$

the core behaviour is completely specified once k_0^{∞} at $t = 0$ is known. This is determined by the loading pattern.

Reloading

At End of Cycle, four year old bundles are discharged, the other bundles are shifted. So, each bundles has its own 4-year *trajectory* in the core. This is described with the variables

$$x_{i,\ell,m} = \begin{cases} 1 & \text{if position } i \text{ contains the bundle} \\ & \quad \text{at age } \ell \text{ from trajectory } m; \\ 0 & \text{otherwise} \end{cases}$$

The assignment constraints are

$$\sum_{\ell} \sum_{m} x_{i,\ell,m} = 1, \quad \forall i$$

$$\sum_{i} x_{i,\ell,m} = 1, \quad \forall \ell, m$$

The reloading is described by

$$k_{i,1}^{\infty} = \sum_{m} x_{i,1,m} \; k^{fresh} + \sum_{\ell>1} \sum_{m} \sum_{j} x_{i,\ell,m} x_{j,\ell-1,m} \; k_{j,EoC}^{\infty}, \quad \forall i.$$

Complete Model

- The assignment constraints:

$$\begin{aligned} \textstyle\sum_\ell \sum_m x_{i,\ell,m} &= 1, \quad \forall i \\ \textstyle\sum_i x_{i,\ell,m} &= 1, \quad \forall \ell, m \end{aligned}$$

- The reloading operation:

$$k_{i,1}^{\infty} = \sum_m x_{i,1,m}\ k^{fresh} + \sum_{\ell>1} \sum_m x_{i,\ell,m} \sum_j x_{j,\ell-1,m}\ k_{j,EoC}, \quad \forall i$$

LOAD

- Core evolution during the cycle:

$$\begin{aligned} k_t^{eff} \phi_t &= G(k_t \cdot \phi_t) && \forall t \\ k_t^T \phi_t &= 1 && \forall t \\ k_{t+1} &= k_t - \alpha \Delta t\, k_t \cdot \phi_t && \forall t \end{aligned}$$

Evaluate

- Safety limitation and Objective:

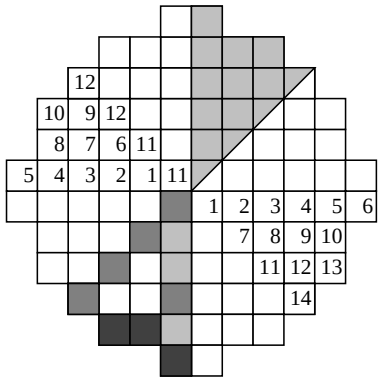
$$k_{i,t} \phi_{i,t} \leq f^{lim} \qquad \forall i, t$$

$$\max \quad k_{EoC}^{eff}$$

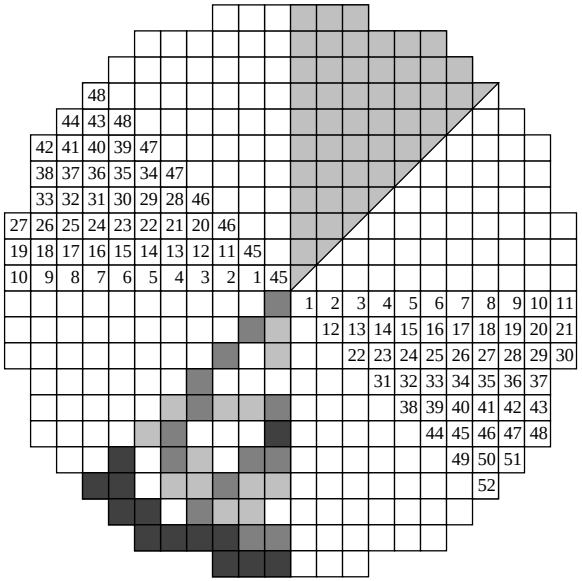
Assumptions

- 2D model, one discretization point per bundle;
- Octane symmetry, with 1/2 nodes on diagonal;
- Homogeneous bundles, stationary reload cycle, no burnable poison.

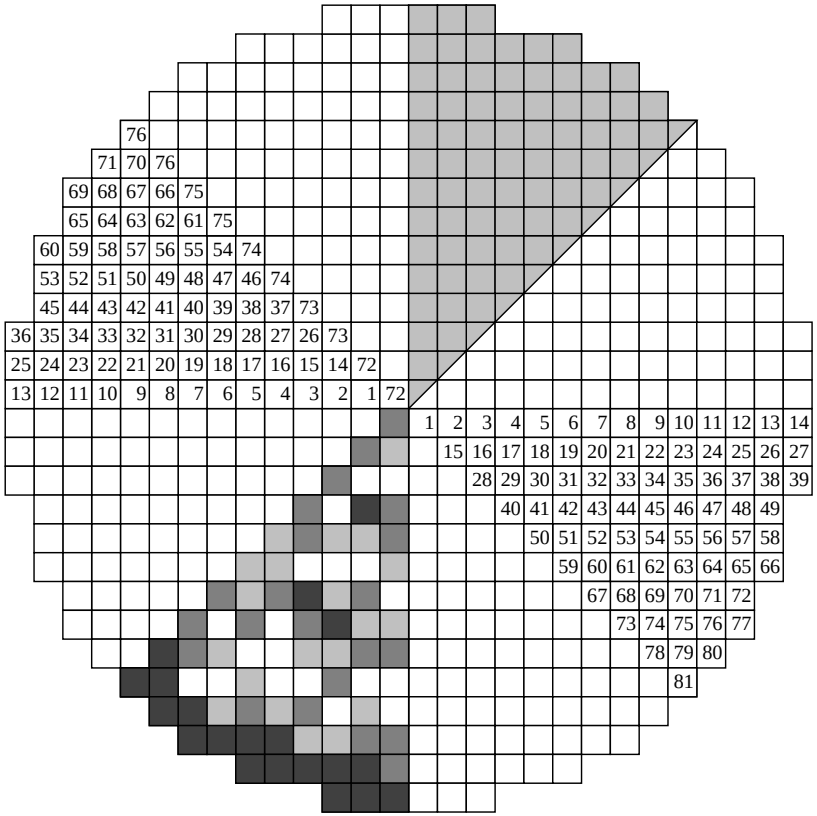
Small core



Large core



Huge core



Solution Methods

- Literature, Practice:
  - Simulated annealing;
  - Genetic algorithms;
  - Pairwise interchange.
- Our work:
  - Nonlinear mixed-integer optimization.
    Motivation:
    * Computation times for local search methods are very large;
    * Model extensions require continuous optimization;
    * Explore model structure for MINLO.

Algorithm

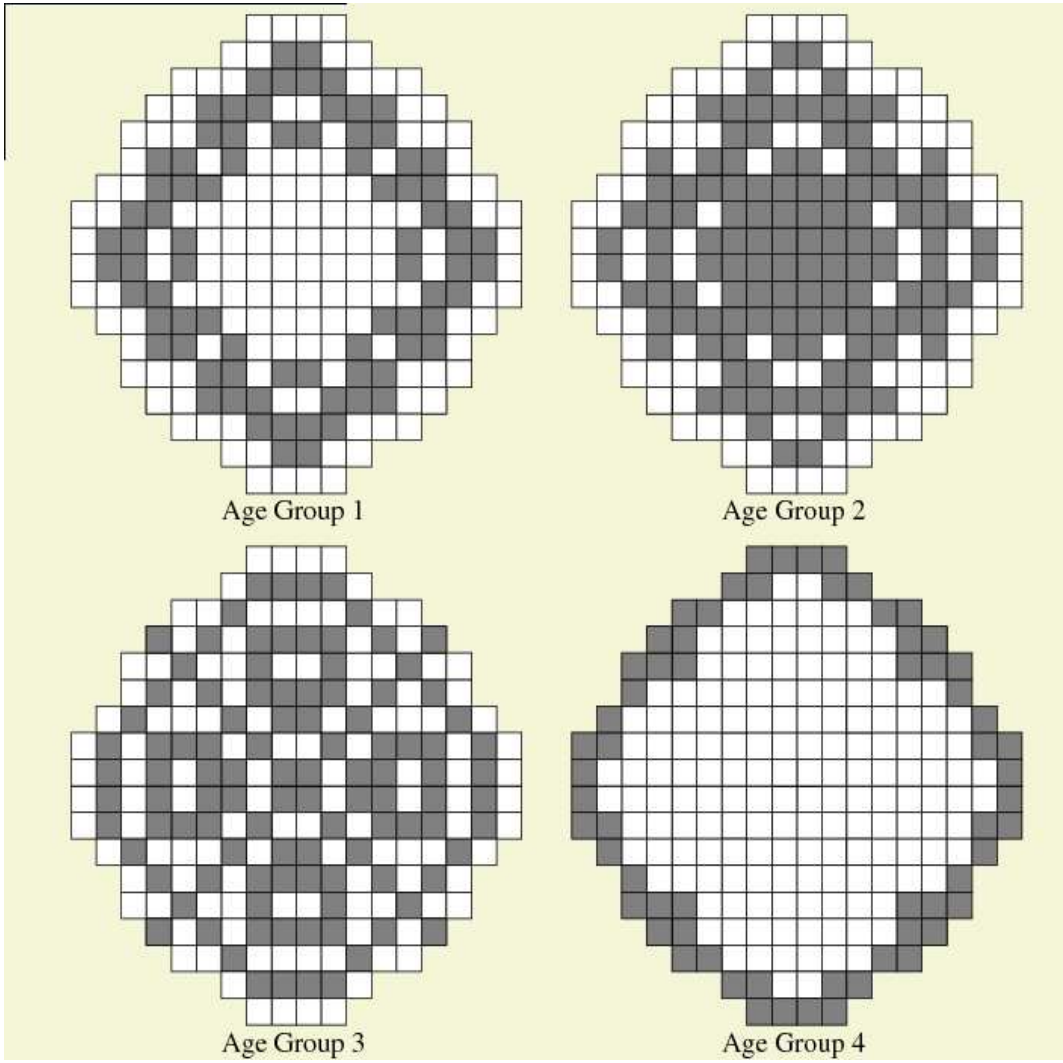
1. Fix starting pattern.
2. Solve kernel equations using power method.
3. Run NLP-solver from this starting pattern.
4. **If** solution is integer **then** go to 7.
5. **For each** possible rounding of the current solution

   (a) Run power method for this fixed pattern.
   (b) If this is the best rounding so far $\rightarrow$ Store.

6. Set current solution = best rounded feasible solution.
7. **If** current solution is integer and feasible **then** local optimum found.
8. (Optionally) Run *local search* starting from the current solution.
9. + **B-and-B, cuts, linearization ...**

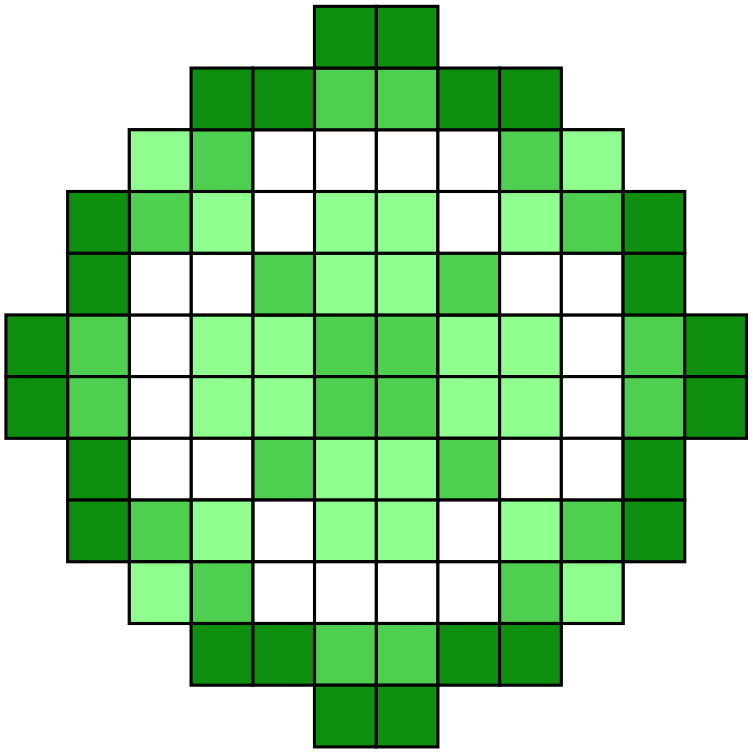
Starting Pattern

- A good starting pattern is extremely important.
- We use a fractional pattern, i.e., bundles are divided over multiple nodes.
- From engineering knowledge, 'most likely positions' for each age are determined. 90% of the age group i is in its corresponding most likely positions.

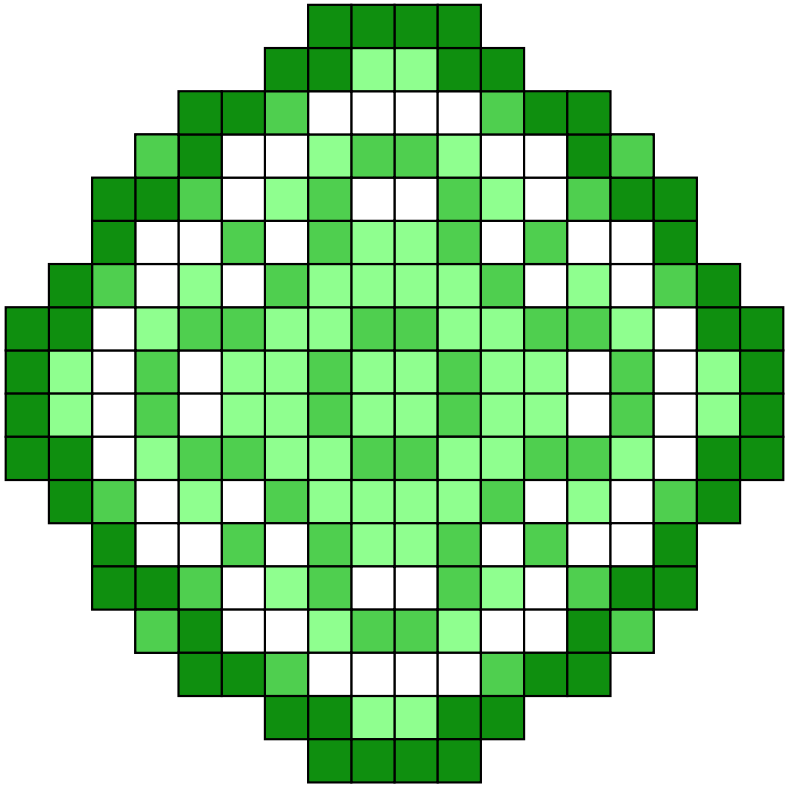


Small and Medium Size Example

Small size solution

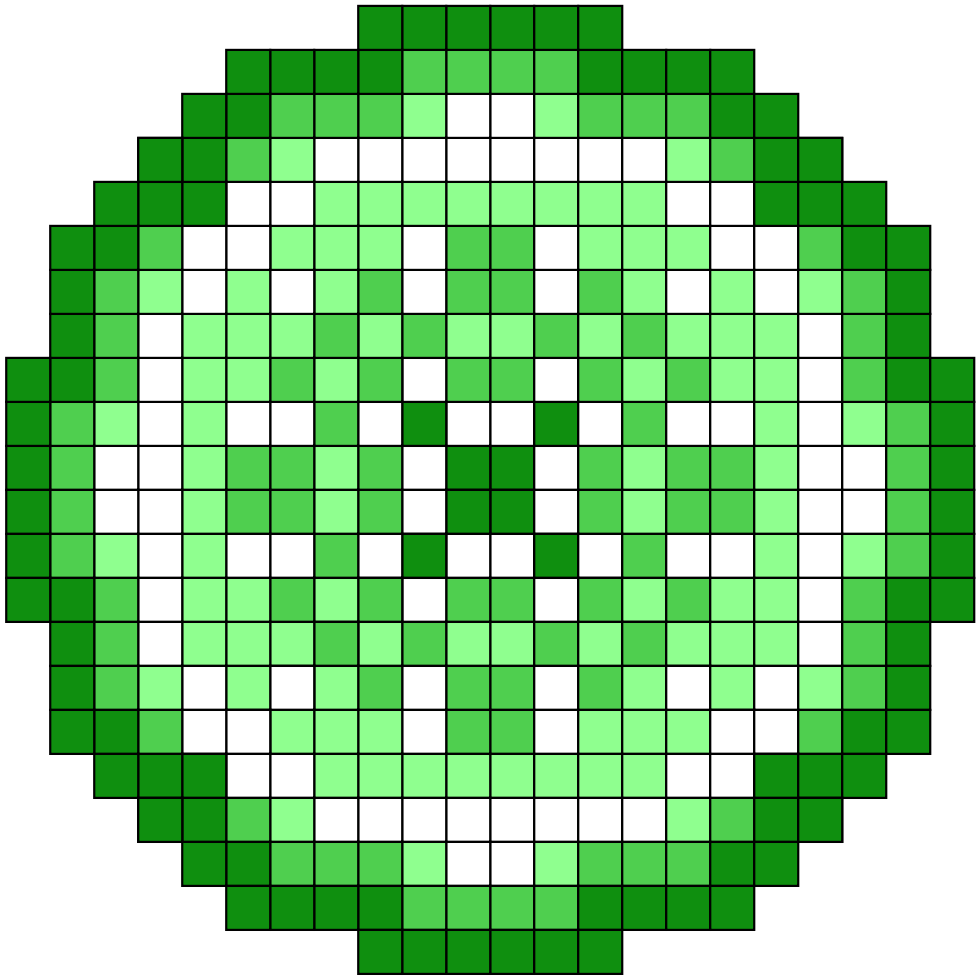


Medium size solution



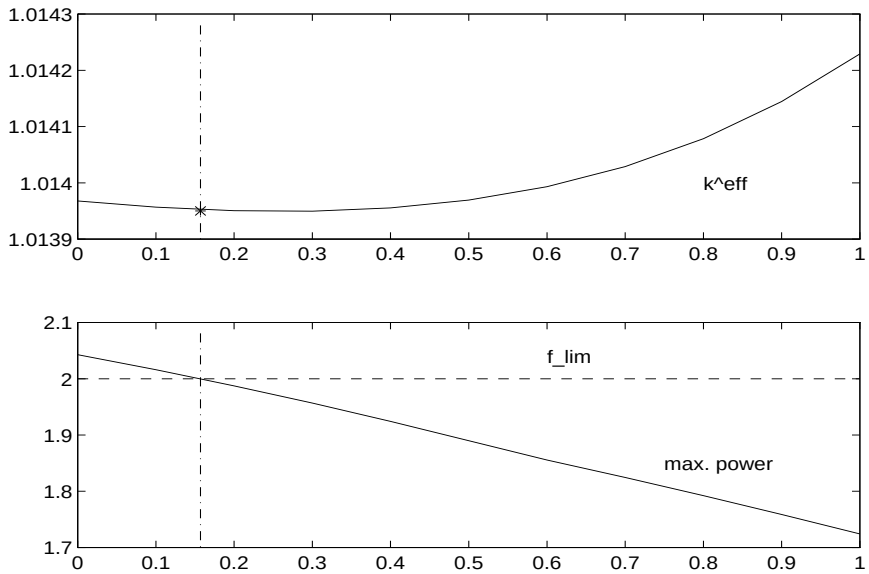
Large Size Example

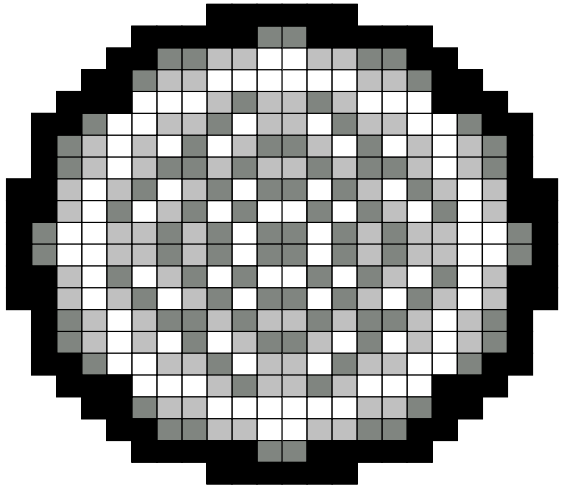
Large size solution

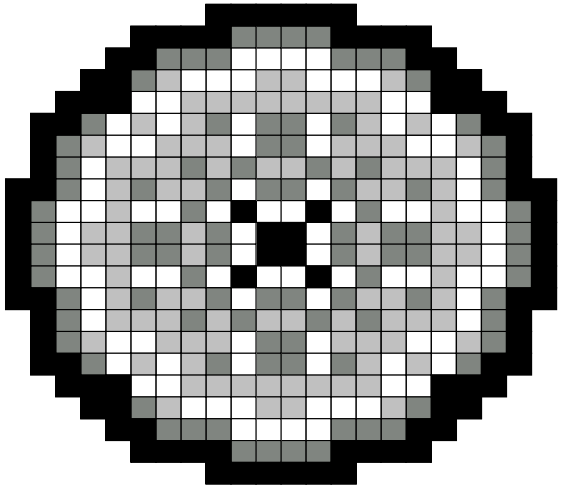


Rounding and Multiple Local Optima

Fractional solution:
objective and powerpeak.
Round to further integer value!



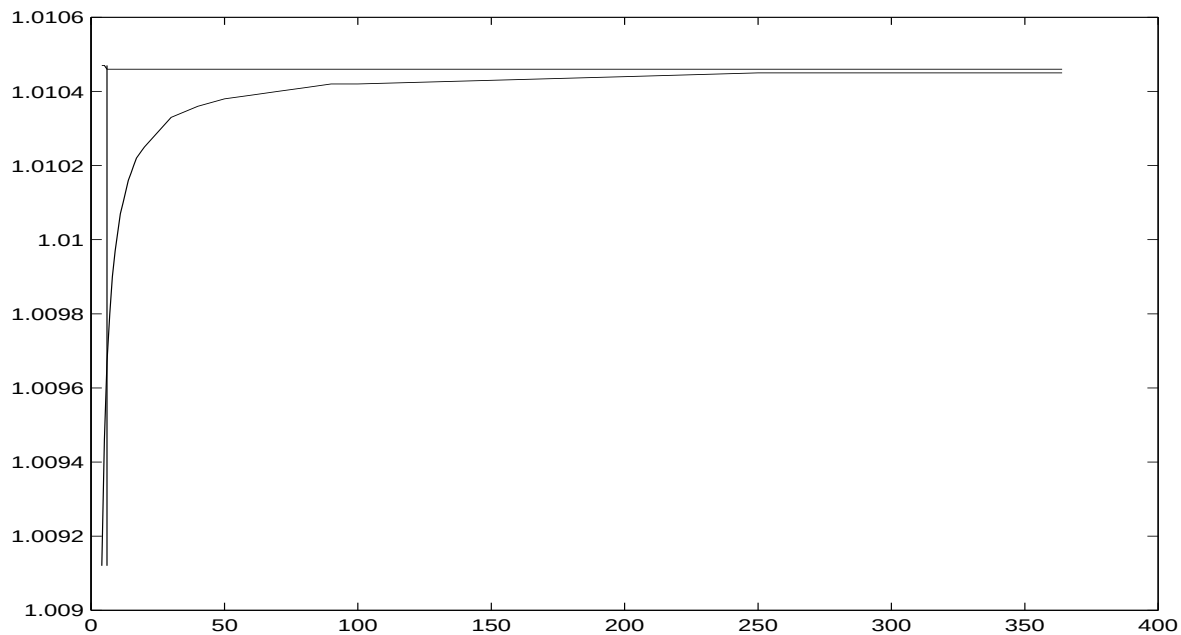




MINLO solution (left) PI solution (right) with with equal objective.

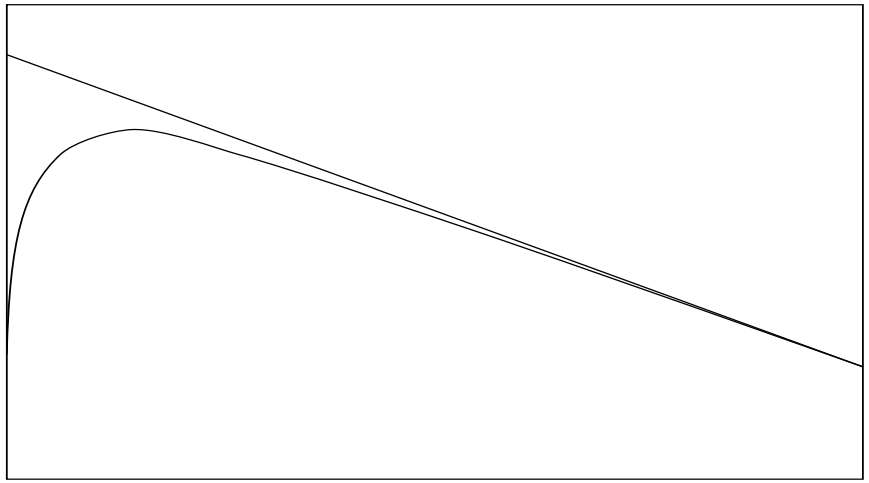
Extension 1: Central Discretization

- **Central vs. Forward time-discretization.** For comparison with methods in practice, we first used forward time-discretization. In those methods, implementation of central discretization requires some predictor-corrector method which is more time-consuming.
- In the NLO context we can use central discretization directly, which is more accurate, results more robust.
- Less time discretization points are needed.



Extension 2: Burnable Poisons

- **Burnable Poisons.** Fresh bundles can cause high local power peaks at Begin of Cycle. This can be avoided by adding neutron absorbing material, that is only active at the beginning.

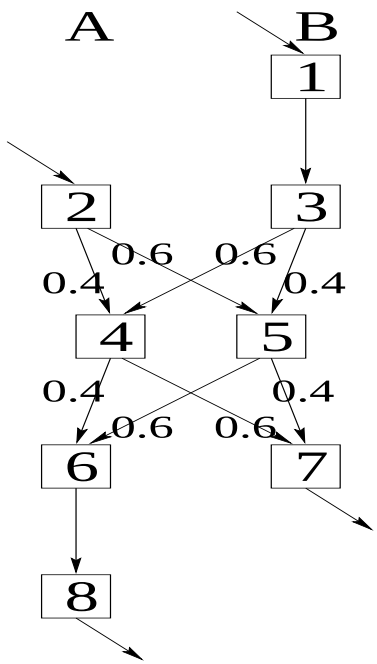


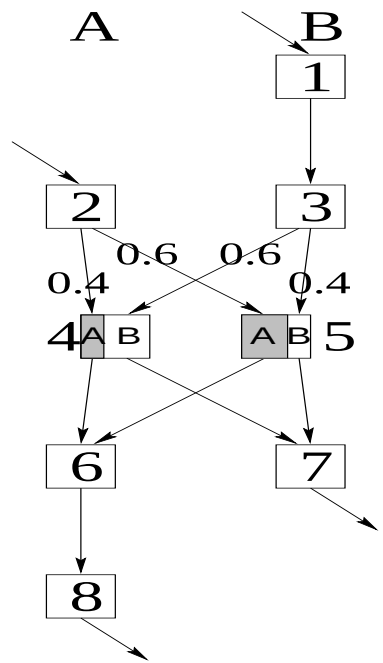
- **Burnup equation:**

$$k_{t+1}^{\infty} - k_t^{\infty} \;=\; -\sigma \Delta t \, b_t(\Delta t) k_t^{\infty} \cdot \phi_t \quad \forall t,$$

- Since the fraction of Burnable Poison can vary continuously, determining the optimal fraction is a continuous optimization problem, that cannot be handled very well with traditional techniques.
  The MINLO approach handles this problem quite well.

Extension 3: More Accurate Model





Use of $x_{i,\ell,m}$: i = node, ℓ = age, m = trajectory.

$x_{2,1,A} = x_{3,2,B} = 1$;

$x_{4,2,A} = x_{5,3,B} = 0.4$;

$x_{4,3,B} = x_{5,2,A} = 0.6$;

$x_{6,3,A} = x_{7,4,B} = 1$.

In the model, node 6 contains a *mixture* of bundles from nodes 2 and 3.

The new model traces the parts to extract the A-parts to 6, B-parts to 7.

Model Extension Continued ...

We must trace the infinite multiplication factor of the fractional parts.

For the original model, this implies that

the reload equation is replaced by

$$\begin{aligned}\widehat{k}^{\infty}_{i,1,m,1} &= x_{i,1,m} k^{fresh}, \ \forall\, i, m \\ \widehat{k}^{\infty}_{i,\ell,m,1} &= x_{i,\ell,m} \sum_{j=1}^{N} \widehat{k}^{\infty}_{j,\ell-1,m,T}, \ \forall\, i,\ \ell,\ m.\end{aligned}$$

The burnup equation is then replaced by

$$\begin{aligned}\widehat{k}^{\infty}_{i,\ell,m,t+1} &= \widehat{k}^{\infty}_{i,\ell,m,t} - \alpha \Delta t\, \widehat{k}^{\infty}_{i,\ell,m,t+1} \phi_{i,t}, \\ &\qquad \forall\, i,\ \ell, \forall\, m,\ t.\end{aligned}$$

Finally, we have to give the relation between all those separate $\widehat{k}^{\infty}_{i,l,m,t}$ variables and the node-average $k^{\infty}_{i,t}$:

$$k^{\infty}_{i,t} = \sum_{\ell=1}^{L} \sum_{m=1}^{M} \widehat{k}^{\infty}_{i,\ell,m,t} \ \forall i,\ t.$$

Results with Burnable Poison

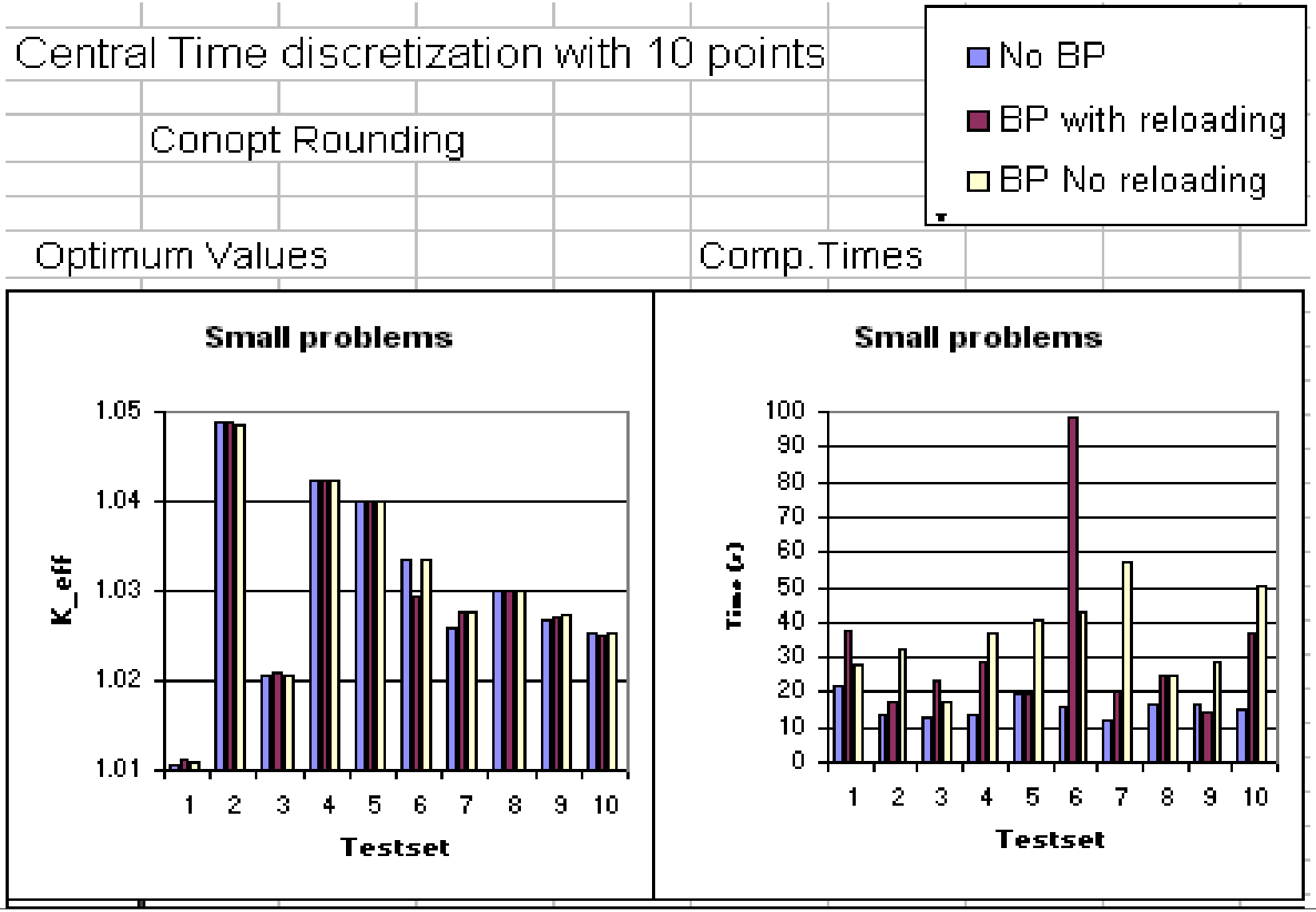
The small test set.

Central Time discretization with 10 points

Conopt Rounding

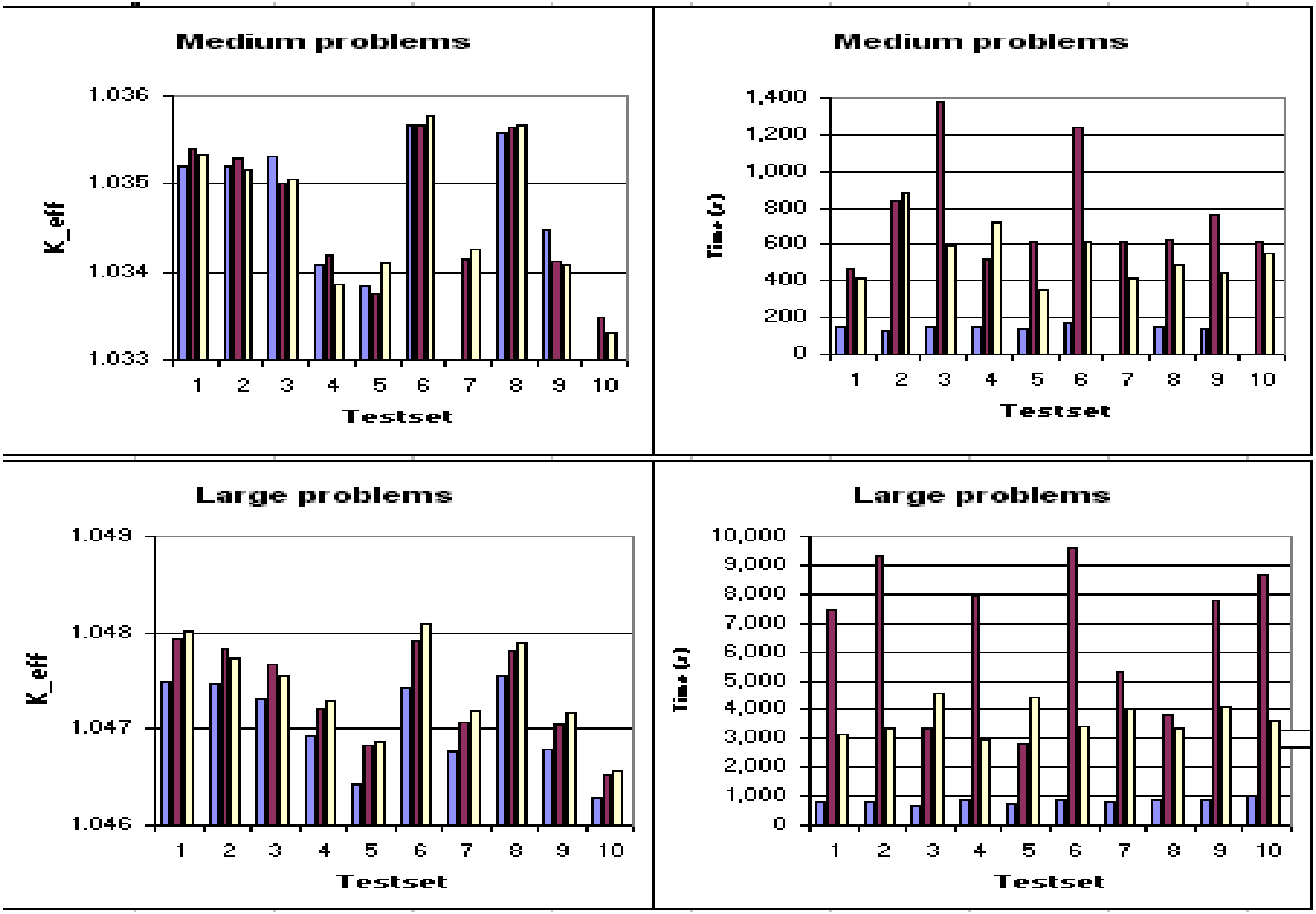
Optimum Values

Comp.Times



Results with Burnable Poison

The medium and the large problems.



Conclusions

- Mixed integer nonlinear optimization has much potential in solving this type of complex problems.
- Good starting points are especially important for good solutions.
- Although for this specific problem, MINLO works well, many integer solutions turn out to be local optima.
- Combination of methods seems to be the best possible way to obtain high quality results in reasonable solution times.
- Using MINLO enables the use of more accurate discretizations, and also enables the develop more complex, realistic models that require continuous decision variables.
- The rounding procedure can be improved by using problem-specific cutting procedures, e.g., age cuts.

Further Work

1. Develop dedicated MINLO solver.
2. Model the rotation of bundles.
3. Model the use of control rods.
4. Model the 3D spacial core.
5. Model non-stationary operation.
6. Model 3D continuous operation, such as the CANDU reactors.
7. **Challenges:**
   - Items 2-5 each increase model size by an order of magnitude.
   - Optimizing continues operation requires fast solution time.

References

- A.J. Quist, R. van Geemert, J.E. Hoogenboom, T. Illés, E. de Klerk, C. Roos and T. Terlaky (1997).
  Application of Nonlinear Optimization to Reactor Core Fuel Reloading (1999). *Annals of Nuclear Energy* 26. 423–448.
- A.J. Quist, R. van Geemert, J.E. Hoogenboom, T. Illés, E. de Klerk, C. Roos and T. Terlaky (1999).
  Finding Optimal Nuclear Reactor Core Reload Patterns Using Nonlinear Optimization and Search Heuristics. *Engineering Optimization* 32. 143-176.
- A.J. Quist, C. Roos, R. van Geemert and J.E. Hoogenboom (2001). Reloading Nuclear Reactor Fuel Using Mixed-Integer Nonlinear Optimization. *Optimization and Engineering* 2(3). 251–276.
- A.J. Quist (2000). *Application of Mathematical Optimization Techniques to Nuclear Reactor Reload Pattern Design*. Ph.D. Thesis. T.U. Delft. The Netherlands.